Positivity and Integrability

(Mathematical Physics at the FU-Berlin)

To Michael Karowski and Robert Schrader on the occasion of their 65th birthday

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Abstract

Based on past contributions by Robert Schrader and Michael Karowski
I review the problem of existence of interacting quantum field theory and
present recent ideas and results on rigorous constructions.

1 Historical remarks

The title of this essay is identical to that of a small conference at the FU-Berlin in honor of Michael Karowski and Robert Schrader at the occasion of their sixty-fifth birthday. The history of mathematical physics and quantum field theory at the FU-Berlin, a university which was founded at the beginning of the cold war, is to a good part characterized by "positivity and integrability" [1].

Both of my colleagues joined the FU theory group in the first half of the 70s shortly after I moved there. Robert Schrader arrived after his important contribution [2] to the birth of Euclidean field theory whose proper mathematical formulation he initiated together with Konrad Osterwalder while working at Harvard university under the guidance of Arthur Jaffe; Michael Karowski came from Hamburg where he finished his thesis under Harry Lehmann. Whereas Robert, after his arrival in Berlin, was still in the midst of finishing up the work with he begun with Konrad Osterwalder at Harvard [3], Michael was looking for new challenging post-doc problems outside his thesis work. At that time Dashen, Hasslacher and Neveu [4] (DHN) had just published their observations on the conjectured exactness of the quasiclassical particle spectrum of certain 2-dimensional models. There were some theoretical indications [5] and numerical checks [6] pointing to a purely elastic S-matrix in those apparently integrable models which were strongly suggestive of an explanation in the (at that time already discredited) S-matrix bootstrap setting, but now within a more special

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context of factorizing elastic S-matrices. It was already clear at that time that such a structural property is only consistent in low spacetime dimensions. In the hands of Michael Karowski and a group of enthusiastic collaborators (B. Berg, H-J. Thun, T.T. Truong, P. Weisz) the S-matrix principles based on crossing, analyticity and unitarity behind these (at that time still experimental mathematical) observations were adapted to two-dimensional purely elastic 2-particle scattering. The findings were published in a joint paper [7] which together with a second paper on the subject of formfactors [8] associated with those factorizing bootstrap S-matrices became the analytic basis for systematic model constructions of quantum field theories based on the bootstrap-formfactor program. The aspect of integrability of these models was verified by constructing a complete set of conserved currents. During this time Robert exploited the new Euclidean framework in order to obtain a constructive control of models. For this purpose he had to use a more restrictive sufficient criterion which limited the class of models to those whose short distance behavior is close to that of free fields.which turned out to be possible in 2-dim. QFT.

All these developments took place in a city which was the most eastern outpost of the western world and for this reason played the role of a show-window of capitalism[1] and liberty which together with its geographic isolation contributed to its well-maintained infrastructure and high quality of life with poverty and social deprivation being virtually unknown. The relative isolation of the city was moderated by a lush funding for inviting guests and there was a very large number of internationally known visitors far beyond the list of international collaborators which each of us attracted after joining the FU faculty.

From those early papers of Michael Karowski and co-workers it became clear that some of the far out speculative conjectures of the Californian (Chew-Stapp...) S-matix bootstrap ideas on uniqueness [2] were without foundation; to the contrary, far from being a unique characterization of a a theory of everything (TOE), the two-dimensional scheme of factorizing S-matrices led to a rich classification of two-dimensional QFT which contained besides the mentioned DHN models many others of physical interest. These new non-perturbative methods attracted a lot of attention and the classification of factorizing S-matrices and the construction of associated integrable models of QFT has remained a fascinating area of QFT ever since.

Through my scientific contacts with Jorge Andre Swieca in Brazil (we both were research associates under Rudolf Haag at the university of Illinois) this line of research took roots at the USP in Sao Paulo and other Brazilian universities and via students of Swieca and Koeberle it led to the formation of a whole group of researchers (E. Abdalla, F. Alcaraz, V. Kurak, E. Marino,..) and

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[1]The capitalism of the cold war was the traditional one i.e. of a different kind as today. These days it is often referred to as the "Rhenish capitalism" in German publications in order to distinguish it from the much more US version..

[2]The S-matrix bootstrap at that time was propagandized as a new theory of everything (apart from gravity). As we all know this was neither the first (it was preceded by Heisenberg's "nonlinear spinor theory") neither the last time. But whereas the old attempts ended in a natural death, the more modern versions are still (not unlike the legendary flying dutchman) circeling over our heads in search of a physical landing place.

also influenced others who nowadays are important members of the Brazilian theoretical physics community.

Besides these two groups which vigorously pursued these new ideas about an S-matrix based construction of low dimensional field theories, there was another independent line with similar aims but stronger emphasis on algebraic structures; this was pursued at the Landau Institute in Moscow (the Zamolodchikov brothers and others [10]) as well as at the Steklov Institute in St. Petersburg (L. Faddeev, F. Smirnov and others [11][12]). There were many scientific exchanges especially with the people from the Steklov institute.

Unfortunately the FU work (this applies in some lesser degree also to Schrader's contributions) did not generate the interests which it deserved within the quantum field theoretical establishment in Germany (Lehmann[3],Symanzik, Zimmermann and Haag), and without this support this became an uphill struggle for the FU group. Several of the highly gifted members of the young FU research group had to end their academic careers; when the recognition of their achievements outside of Germany had a positive feedback within it was already too late[4].

After the fall of the Berlin wall the situation with respect to fundamental QFT research in Berlin worsened. The physics department of the Humboldt University was restructured solely by "Wessies" (to use the colloquial Berlin slang of those days which has survived up to the present); the home-grown community in QFT had no say and was not asked for advice. Considering the importance of historical continuity in particle theory and the intellectual damage caused by political interference, it is not surprising that the consequences of those negative influences have left their mark and will become even more evident after two of the last FU-QFT innovators go into retirement. Outside attempts to fill fundamental quantum field theoretical research with the passing flow of theoretical fashions did not work.

But this article is not primarily an essay of past achievements of two of my colleagues, nor about the history of QFT and the ups ans downs of mathematical physics in Berlin. I rather prefer to demonstrate the relevance of their past innovations by convincing the reader that the legacy of their ideas constitute still an important part of the ongoing scientific dialogue.

In order to be able to do this, I first have to recall some of the pre-electronic conceptual advances which got lost or failed to get passed in the proper way to the younger generation. A good illustration of such loss of knowledge as a result of oversimplifications and distortions caused by globalized fashions is the fate suffered by the *Euclidean method.* The work of Osterwalder and Schrader and prior contributors started with a very subtle and powerful problematization of what is behind the so-called Wick rotation. These days I sometimes find myself

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[3]There is some irony in the fact that Lehmann at the end of his life became actively interested in low dimensional theories. But at that time his influence on particle physics in Germany was already declining.

[4]One outstanding younger member of the group was H. J. Thun whose joint work with Sidney Coleman [9] on the nature of higher poles in the S-matrix became a standard reference. Despite his impressive start there was no place for his academic carrier in Germany.

attending talks in which the speaker did not bother to explain whether he is in the setting of real time QFT or in the Euclidean setting; a related question during such a talk usually raises the speakers eyebrows because to him in his intellectual state of innocence it is evident that one can pass from one to the other without being bogged down by the need of justification. The rapid rate in which fundamental knowledge gets lost or substituted by universal phrases is one of the symptoms of a deep crisis in particle physics.

Indeed the Euclidean theory associated with certain families of real time QFTs is a subject whose subtle and restrictive nature has been lost in many contemporary publications as a result of the "banalization" of the Wick rotation (for some pertinent critical remarks referred to in [13]). The mere presence of analyticity linking real with imaginary (Euclidean) time without checking the validity of the subtle reflection positivity (which is necessary to derive the real time spacelike commutativity as well as the Hilbert space structure) is not of much physical use.

In times of lack of guidance from experiments the most reasonable strategy is to press ahead with the intrinsic logic of the existing framework, using the strong guidance of the past principles and concepts rather than paying too much attention to the formalism which was used to for their implementation. In a previous particle physics crisis, namely that of the ultraviolet divergencies of QFT, it was precisely this attitude and not the many wild speculations in the decade before renormalization theory, which finally led to the amazing progress; in fact renormalization theory was probably the most *conservative affirmation of the underlying causality and spectral principles* of Jordan's "Quantelung der Wellenfelder". What was however radically different was their new mathematical and conceptual implementation.

Although this impressive progress made QFT what it is today, namely the most successful physical theory of all times up to this date, it is still suffering from one defect which sets it apart from any other area of theoretical physics. Whereas in other areas the construction of models preceded the presentation of a setting of axioms (which extract the shared principles underlying the explicit constructions), things unfortunately did not work this way in QFT. The reason was that the perturbative constructions (unlike say in mechanics, astronomy and quantum mechanics) did not come with mathematical assertions concerning their convergence and estimates of errors. These aspects were not only missing in Feynman's perturbative (and any differently formulated) approach, but it became increasingly clear that all these series were at best only asymptotically converging (which unfortunately is a property which does not reveal anything about the mathematical existence).

This led the birth of an axiomatic framework[5] which was followed by constructive QFT. Measured in terms of the complexity of the problem, the results

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[5]This terminology has often been misunderstood. It has a completely different connotation than say axiomatics of mechanics or thermodynamics, since it results from the realization that it is much harder to do a credible computation for a concrete model than it is to understand joint structural properties of a whole class of models (as long as the question of existence is ignored).

and the methods by which they were obtained are impressive [25]. There was also a certain amount of elegance which clearly came from a very clever use of Euclideanization and/or algebraic properties. Conceptually these methods followed closely the physical ideas underlying renormalized perturbation theory. As in the perturbative approach the main objects to be controlled are correlation functions of pointlike fields, with free fields and their Fock space still playing an important auxiliary role. An important technical step was to establish a measure-theoretical interpretation to the interaction-polynomial relativ to the free field measure..As a result of methodical limitations it was virtually impossible to go beyond the very restrictive superrenormalizability requirement and to incorporate real life models (as e.g. the Standard Model). For reasons of a certain imbalance between an unwieldy mathematical formalism and the few (and mostly anyhow expected) results besides the control of existenc, the *constructive approach* did not enter standard textbooks but rather remained in the form of reviews and monographies; expert cynics even sometimes referred to it (in particular by H. Lehmann) as "destructive QFT". Most practitioners of QFT do not mention this problem or raise their students with the palliative advice that since QFT is not credible for very short distances, the existence problem is somewhat academic; but unfortunately without giving the slightest hint how in physics (i.e. outside of politics) problems can be solved by enlarging them. Recently there has been some renewed interest in a variant of this method which is based on the hope that some progress on the functional analytic control [14] of time-dependent Hamiltonian problems may extend the mathematical range (of a more modern algebraic formulation) of the Bogoliubov S-operator approach beyond what had been already achieved in [15].

For a number of years I have entertained the idea that one is struggling here against a birth-defect of QFT whose effects can be only removed by a some radical conceptional engineering. I am referring here to the that *classical paralellism* called Lagrangian quantization by which Pascual Jordan found the "Quantelung der Wellenfelder"[6]. What is most amazing is the fact that only two years after his discovery he apparently became worried about this kind of quantization not being really intrinsic to quantum physics (at that time, shortly after computing the Jordan-Pauli commutatot function, he also could have been already aware that pointlike quantum fields as obtained from Lagrangian quantization are singular objects besides lacking intrinsicness). Although we do not know his precise motivation, as the main plenary speaker at the first post QFT big international conference 1929 in Kharkov [17] (probably the last international sympsium in which German was the conference language) he pleaded to look for a new access to QFT which avoids such "classical crutches" (klassische Kruecken) but without proposing a way to implement this idea. Apart from

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[6]His peer Max Born with Heisenberg's support limited Jordan's obsession with quantizing also structures beyond mechanics by banning his calculations to the last section of the joint work. Darrigol [16] reports that when Jordan received Schroedinger's results he already had what was later called a second quantized version. For his radical viewpoint it was apparently sufficient that a structure could be fitted into a classical framework and not whether it was actually part of classical physics.

Wigner's isolated representaion-theoretical approach to relativistic particles in 1939 (which only in the late 50's became known to a broader public through the work of Wightman and Haag), the first entirely intrinsic setting which avoided field coordinatizations (non-intrinsic generators of spacetime-indexed algebras) was the algebraic approach by Haag (with some mathematical ideas concerning operator algebras taken from Irving Segal), later known as the Haag-Kastler approach. But in some sense the baby was thrown out with the bath water because the conceptual precision was not matched by any calculational implementation. This was at least the situation before the modular theory of operators algebras was incorporated in order to place local quantum physics on a more constructive course. At this point Karowski's contribution (and more generally the meanwhile extensive literature on the classification and construction of factorizing model) enters; these explicit and nontrivial model constructions represent presently the most valuable theoretical laboratory for the new constructive ideas based on modular theory; they have led to new results concerning the proof of existence of certain strictly renormalizable models to which we will return later. These new developments suggest that it rather improbable to win a prize for the solution of gauge theory as an isolated subject without revolutionizing the whole of local quantum physics.

The main motivations for writing this essay is to point out that both the issue of Euclideanization and the bootstrap-formfactor approach are both by no means mature closed subjects; rather they are illustrative examples which shows that QFT despite its age is still very far from its closure.

The article is organized as follows. The next section reviews some of the ideas which led to the Osterwalder-Schrader results and their role in strengthening the constructive approach to QFT. In the same section I also sketch the more recent framework of modular localization. Its analyticity aspects derive from the domain properties of a certain unbounded operator which characterizes localization aspects of operator subalgebra; unlike the Bargmann-Hall-Wightman analyticity of correlation functions of pointlike fields (operator-valued distributions) which constitute an important aspect of the Euclidean approach, the Tomita-Takesaki modular theory does not refer to individual operators but rather encodes joint properties of operators which are members of an operator algebra. Euclideanization aspects of modular theory are presented in section 4. The third section sketches the notion of integrability in QFT which is synonemous with factorization of the S-matrix, but in order to maintain a unified conceptual line I deviate from the historical path and present factorizing models in the modular setting. In the last section I list some open problems related to the theme of this essay.

2   Positivity and Euclideanization

After the rehabilitation of the divergence-ridden QFT in the form of renormalized perturbative quantum electrodynamics, and after the subsequent conceptual advances in the understanding of the particle-field relation through scat-

tering theory [18][19], the idea gained ground that the importance of QFT for particle physics can be significantly enlarged by understanding more about its model-independent structural properties beyond perturbation theory. The first such general setting was that by Wightman [20]. In harmony with the increasing importance of analytic properties which entered the setting of scattering theory through the particle physics adaptation of the optical Kramers-Kronig dispersion relations, special emphasis was placed on the study of analytically continued correlation functions. These investigations started from the *positive energy momentum* spectrum (expressing the presence of a stable ground state, the vacuum) and the *Lorentz-covariant transformation* properties as well as *locality* (in the form of (anti)commutators vanishing for spacelike distances). This, led to an extension of the original tube domain resulting from the positive energy-momentum spectrum to the Bargmann-Hall-Wightman region of analyticity and its extension by locality. The continued correlation functions turned out to be analytic and uni-valued in the resulting "permuted extended tube" region [20].

Already before these mathematical results the Euclidean region, which resulted by letting the time component be pure imaginary, attracted the attention of Schwinger since in his formalism it led to some computational simplification. The next step was taken by Symanzik [21] who observed from his functional integral manipulations that the analytic continuation to imaginary times (Wick rotation) for bosonic theories highlighted a positivity which was well-known from the continuous setting of statistical mechanics. Nelson [22] succeeded to remove the somewhat formal aspects and achieved a perfect placement into a mathematically rigorous setting of an autonomous stochastic Euclidean field theory (EFT) in which the most important conceptual structure was the Markov property. Guerra [23] applied this new setting to control the vacuum energy coming from certain polynomial interactions in bosonic 2-dimensional QFT and emphasized its usefulness in establishing the existence of the thermodynamic limit. This Euclidean field formalism was limited to fields with a canonical short distance behavior; but even in this limited setting *composite fields* with worse short distance behavior permit no natural incorporation into this probabilistic setting. The limitation was intimately related to the property of Nelson-Symanzik positivity, which basically is the kind of positivity which Schwinger functions should obey if they were to describe a (continuous) stochastic classical mechanics.

The breakthrough for the understanding of Euclideanization of the general situation in QFT was achieved in the work of Osterwalder and Schrader [2]. They had to sacrifice the stochastic interpretation of EFT which was then substituted by a certain reflection positivity condition as well a growth condition on n-point Schwinger functions for n → ∞. If one is less ambitious and only asks for a sufficient condition on Schwinger functions, one obtains a formulation which turns out to be quite useful for controlling the existence for certain low-dimensional QFTs.

One reason why the constructive control of higher dimensional QFTs present a serious obstacle is that the reflection positivity does not harmonize well with the idea of (Euclidean invariant) ultraviolet cutoffs. For this reason one encoun-

ters serious difficulties with ultraviolet cutoffs in a functional integral setting; in general one does not even know whether such a cutoff is consistent with the quantum theory setting; not to mention all the other requirements as e.g. cluster properties, asymptotic scattering limits etc. which one needs to maintain the physical interpretation of a theory. Of course cutoff versions are strictly auxiliary constructs and as such may violate such properties, but then the control of the cutoff-removal becomes a hairy problem. On the other hand it is possible to formulate the process of O-S Euclideanization by starting from the more algebraic setting of AQFT which avoids the use of (necessarily singular) point-like field coordinatisations; in this case one has problems to specify concrete interactions. A formulation of the algebraic approach in the lattice setting for which the concepts and their mathematical implementation of the O-S euclidianization allow a very simple presentation can be found in [24].

One property within the Nelson-Symanzik setting which turned out to be extremely useful in controlling the removal of infrared regularizations (thermodynamic limit) is the Euclidean spacetime duality. This *Nelson-Symanzik duality* is suggested by the formal use of the Feynman-Kac Euclidean functional integral representation. Let us consider thermal correlation functions at inverse temperature β for a 2-dimensional enclosed in a periodic box (rather interval). The KMS condition for the correlation functions at imaginary times reduces to a β-periodicity property. Since the Euclidean functional representation treats space and time on equal footing, the duality under a change of x and t_E accompanied by an exchange of the box- with the thermal- periodicity is obvious. The mathematical physics derivation of this result can be found in a recent paper [26]. In the last section we will use this property as an analogy of the chiral *temperature duality*. A model-independent systematic adaptation of the O-S Euclideanization to the imaginary time thermal setting can be found in a recent review paper [35].

In the remainder of this section I will recall the modular localization setting for the convenience of the reader. This is a preparatory step for the content of the last section. The salient properties of the modular aspects of QFT can be summarized as follows [27].

- Modular localization is an adaptation of the modular Tomita-Takesaki theory in the setting of operator algebras. The analytic properties are not associated to local covariant fields but rather to the operator algebra $\mathcal{A}(\mathcal{O})$ which is associated with smeared fields if one limits the test function supports to a fixed spacetime region $\mathcal{O}$. Modular theory [28] is based on the idea that one learns a lot about operator algebras by studying the unbounded antilinear and (as it turns out) closed operator S defined as

$$SA\Omega = A^*\Omega, \ A \in \mathcal{A} \tag{1}$$

  where Ω is a cyclic (i.e. $\mathcal{A}\Omega$ is dense in H) and separating (there is no nontrivial $A \in \mathcal{A}$ which annihilates Ω). Interesting properties arise from its polar decomposition which is traditionally written as

$$S = J\Delta^{\frac{1}{2}} \tag{2}$$

The resulting unbounded positive operator Δ generates via its one-parametric unitary group Δ^{it} a modular automorphism group of $\mathcal{A}$ and the "angular" part J, the so-called Tomita conjugation, is an antiunitary involution which maps the operator algebra into its commutant $\mathcal{A}'$

$$\sigma_t(\mathcal{A}) \equiv \Delta^{it} \mathcal{A} \Delta^{-it} \subset \mathcal{A}, \; J\mathcal{A}J = \mathcal{A}' \tag{3}$$

where I used a condensed notation using $\mathcal{A}$ as a short hand for its individual operators $A \in \mathcal{A}$. The crucial property of the Tomita S which is behind all this algebraic richness is the fact that S is "transparent" in the sense that $dom S = ran S = dom \Delta^{\frac{1}{2}}$, $S^2 = 1$ on $dom S$. I am not aware of the existence of such unusual (not even in Reed-Simon) operators outside modular theory. This theory begins to unfold its magic power within QFT[7] once one realizes (as was first done by Bisognano and Wichmann [29]) that not only any pair $(\mathcal{A}(\mathcal{O}), \Omega = vacuum)$ with a nontrivial spacelike disjoint $\mathcal{O}'$ is "standard" in the sense of modular theory but even more: for the standard pair $(\mathcal{A}(\mathcal{W}), \Omega)$ with $\mathcal{W}$ a wedge region, the modular group acts as the unique W-preserving Lorentz-boost and the Tomita reflection is (up to a rotation which depends on the choice of W) equal to the physically extremely significant TCP symmetry. Whereas the unitary $\Delta_{\mathcal{W}}^{it}$ is "kinematical" i.e. determined once the representation theory of the Poincaré group (the spectrum of particles) is known, the J_W contains profound dynamical information. If we assume that we are in the standard LSZ setting of scattering theory[8] then the J of an interacting theory is connected by its interaction-free asymptotic counterpart J_0 through the scattering matrix

$$J = S_{scat} J_0 \tag{4}$$

i.e. whereas in the interaction-free case the modular data for the wedge algebra are constructed in terms of the relevant representation of the Poincaré group, the presence of interactions enriches the modular theory of wedge algebras through the S-matrix. For models for which the bootstrap construction of their S-matrix can be separated from the construction of their fields (the factorizing models of the next section) the knowledge of the modular data can be used for their explicit construction. The guiding ides is that knowing the modular data for the wedge algebra uniquely fixes the modular operators for all the other causally complete region. Although there is no geometro-physical interpretation a la Bisognano-Wichmann for the modular objects of smaller causally closed spacetime regions (spacelike cones, double cones), there is no problem in constructing them through the process of algebraic intersections in terms

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[7]Actually the constructive power of the modular approach only began to unfold after a seminal paper by Borchers [30] which led to a flurry of additional remarks [31][32] and finally gave rise to the theory of modular inclusions and modular intersections [33][34].

[8]The LSZ asymptotic convergence of Heisenberg operators towards free (incoming or outgoing) particle operators is guaranteed by spacelike locality and the assumption of gaps which separate the one-particle massive state from the continuum [19].

of wedge algebras (such $\mathcal{O}$ are necessarily causally closed)

$$\mathcal{A}(\mathcal{O}) = \bigcap_{\mathcal{W} \supset \mathcal{O}} \mathcal{A}(\mathcal{W}) \qquad (5)$$

The impressive constructive power of this theory already shows up in its application to the Wigner representation theory of positive energy representations of the Poincaré group. The results obtained by combining Wigner's theory with modular localization go beyond the well-known results of Weinberg on three counts:

- The spatial version of the modular localization method associates string-like localized fields with Wigner's enigmatic family of massless infinite spin representations whereas previous attempts at best showed that these representations are incompatible with point-like localization.

- For massless finite helicity representations (photons, gravitons,...) only the "field strength" whose scale dimension increases with the helicity are pointlike whereas the "potentials" with would-be dimension one turn out to be string-like[9] i.e. pointlike potentials are incompatible with the Wigner representation theory.

- The structural analysis carried out by Buchholz and Fredenhagen on massive theories with a mass gap suggests strongly that the setting of interactions may be significantly enlarged by permitting interactions to possess a string-like localization structure. If one wants to implement this idea in a perturbative setting one needs massive string-localized free fields. The application of modular localization leads to *scalar string-like localized fields* for arbitrary spins.

There is another important representation theoretical result from modular localization for d=1+2 dimensional QFT which according to my best knowledge cannot be derived by any other method. It is well-known that the (abelian in this case) spin in this case can have anomalous values which activates the representation theory of the universal covering $\tilde{P}(3)$ first studied by Bargmann. Combined with the modular localization theory one is able to determine the localized subspaces and a "preemptive" one-particle version of a plektonic spin-statistics theorem. In this case the transition from the Wigner representation to the QFT can however not be done in a functorial way since there is an inherent vacuum polarization related with nontrivial braid-group statistics [36].

It is interesting to compare the setting of modular localization with the O-S euclideanization. The former also leads to analyticity properties and to euclidean aspects but in this case they are not coming from Fourier-transformed support properties and their covariant extension but rather encode domain properties of unbounded operators**.** The connection with analyticity properties and

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[9]In the strict Heisenberg-Wigner spirit of observables one rejects unphysical ghosts (which formally make potentials covariant and pointlike) even though they are only computational catalyzers in order to obtain observables at the end of the computation.

"Euclideanization" is through the KMS-property of $\mathcal{A}(\mathcal{O})$ expectation values in the state implemented by the vector Ω. The defining equation for S shows that all vectors of the form $A\Omega$ are in $dom\Delta^{\frac{1}{2}}$, which means that these vectors $\Delta^{iz}A\Omega$ are analytic in the strip $-Imz < \frac{1}{2}$.

The most attractive and surprising property of this formalism is the encoding of geometry of localization in domain properties (and a forteriori in analyticity) in the sense $S_{\mathcal{O}_1} \subset S_{\mathcal{O}_2}$ if $\mathcal{O}_1 \subset \mathcal{O}_2$ and $S_{\mathcal{O}_1} \subset S^*_{\mathcal{O}_2}$ if $\mathcal{O}_1 >< \mathcal{O}_2$ [38]. Such an intimate relation between domain (and range) properties of unbounded and geometric localization properties is unique in particle physics and is not met anywhere else in physics (this probably explains why it is not treated in books on mathematical-physics methods as Reed-Simon)

In the present stage of development of the modular formalism does not permit a general classification and constructive control in the presence of interactions. As the Euclidean formalism it is limited to certain low dimensional QFT but for quite different reasons. What is interesting is that the families of low dimensional models covered by the two settings are quite different. Whereas for the Euclidean approach the limitation is the traditional one coming from short distance properties, the present limitation of the modular approach has nothing to do with short distance properties of pointlike fields but rather is tied to the existence of generators of wedge algebras $\mathcal{A}(\mathcal{W})$ with simple physical properties, so called tempered vacuum-polarization-free generators (PFG). Some details will be explained in the next section. It turns out that this requirement is equivalent to the S-matrix being factorizing.

The test of existence of a model (which has been defined in terms of its generators for the wedge-localized algebra) in the modular approach is not related to its good short distance behavior, but rather consists in the nontriviality of algebraic intersections.

In the last section I will present a Euclideanization based on modular localization which shows its analogy to the O-S setting.

3   Tempered PFG, integrable QFT, factorizing models

Modular localization offers a surprising way to obtain new insight into field theoretic integrability and the classification of factorizing models. As before we assume the existence of isolated one-particle mass-shells which is sufficient for the validity of scattering theory. The starting point is the following definition which then leads to two theorems.

Definition 1 *A vacuum-**p**olarization-**f**ree-**g**enerator (PFG) of a localized algebra $\mathcal{A}(\mathcal{O})$ is a (generally unbounded) operator $G^{\#}$ affiliate to this algebra which applied to the vacuum creates a one-particle state without vacuum polarization*

admixture

$$G\Omega = one - particle\ vector \qquad (6)$$
$$G^*\Omega = one - particle\ vector \qquad (7)$$

It is clear that a (suitably smeared) free field is a PFG for any free field subalgebra $\mathcal{A}(\mathcal{O})$, but it takes some amount of thinking to see that the inverse also holds i.e. the existence of a PFG for any causally complete subwedge region $\mathcal{O}$ implies that G is a smeared free field and that the superselection-sector generated by G is that of a free field sector. On the other hand the (in/out) particle creation/annihilation operators are affiliated to the global algebra. The wedge region is a very interesting borderline case; the application of modular theory shows that PFGs in interacting theories do exist in that case i.e. in more intuitive physical terms: *the wedge localization is the best compromise between particles and fields in interacting QFTs*. A closer examination reveals that if one demands that PFGs are tempered in the sense that they have domains which are stable under spacetime translations, the S-matrix is necessarily purely elastic [37]. This in turn reduces the possibilities (excluding "free" models with braid group statistics in d=1+2) to d=1+1 dimensional interacting theories and in that case one indeed has the rich class of factorizing S-matrices as illustrative examples.

Theorem 2 *[37]Tempered PFGs are only consistent with purely elastic S-matrices, and (excluding statistics beyond Bosons/Fermions), elasticity and non-triviality are only compatible in d=1+1.*

The crossing property for formfactors excludes connected elastic 3-particle contributions [10] so that the factorizing models actually are the only ones whose wedge algebras are generated by PFGs. This approach culminates in the recognition that the generators of the Zamolodchikov-Faddeev algebras are actually the Fourier transforms of the tempered wedge-localized PFGs; in this way the computational powerful but hitherto (in the LSZ scattering setting) conceptually somewhat elusive Z-F operator algebra acquires a physical spacetime interpretation. Since there are some fine points concerning wedge-localization in the presence of bound states (associated with certain S-matrix poles in the physical rapidity strip) I will for simplicity assume that there are none. For models with a continuous coupling strength (e.g. the Sine-Gordon model) this is achieved by limiting the numerical value of the coupling parameter. Let us further assume that the particle is spinless in the sense of the Lorentz-spin. Then the following theorem holds

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[10]Private communication by Michael Karowski.

Theorem 3 *Let $Z^{\#}(\theta)$ be scalar Z-F operators i.e.*

$$Z(\theta)Z^*(\theta') = S_2(\theta - \theta')Z^*(\theta')Z(\theta) + \delta(\theta - \theta') \qquad (8)$$
$$Z(\theta)Z(\theta') = S_2(\theta' - \theta)Z(\theta')Z(\theta)$$
$$\phi(x) = \frac{1}{\sqrt{2\pi}} \int (e^{ip(\theta)x(\chi)}Z(\theta) + h.c.)d\theta$$

then the non-local fields generate a wedge-localized algebra $\mathcal{A}(\mathcal{W})$ and the co-efficient functions S_2 are the two-particle scattering matrix contributions of a purely elastic factorizing scattering matrix S_{scat}

As already stated in the previous section, the algebras for compact spacetime regions and their pointlike field generators are constructed by forming intersections of wedge algebras. The relevant calculations are very simple in the case of interaction-free fields associated with the various families of positive energy Wigner representations [39][27]. For the case at hand they are slightly more involved, reflecting the fact that although the PFG generators are still on-shell but the creation/annihilation components have a more complicated algebraic structure. There are two strategies to be followed depending on what one wants to achieve.

If the aim is to establish the existence of the model in the algebraic setting, then one must find a structural argument which secures the nontriviality of intersections of wedge algebras associated to causally complete spacetime regions. For the case at hand the property of *modular nuclearity* is sufficient to show nontriviality. There are some recent interesting partial results in this direction by Lechner [40]. Meanwhile there exists a proof which applies to all factorizing models whose S-matrix depends on a coupling strength such that for weak coupling there is no bound state [41].

The underlying physical idea is that the nontriviality is already encoded into the structure of the wedge algebra generators. In particular in d=1+1, a property called *modular nuclearity* of the wedge algebra (referring to the cardinality of phase space degree of freedoms [42]) secures the nontriviality of double cone intersections which is tantamount to the existence of the model in the framework of local quantum physics. Since the proof uses the S-matrix in in an essential way it is not surprising that certain properties which were extremely hard to obtain in the approach based on Euclideanization as the condition of asymptotic completeness, are a quite easy side result of the nontrivial existence arguments.

If on the other hand the aim is to do explicit calculations of observables beyond the S-matrix, then the determination of the formfactor spaces is the right direction to follow. In that case one makes a Glaser-Lehmann-Symanzik-like Ansatz, but instead of expanding the desired localized Heisenberg operator in terms of incoming creation/annihilation operators, ones uses the Z-F operators instead

$$A = \sum \frac{1}{n!} \int_C ... \int_C a_n(\theta_1, ...\theta_n) : Z(\theta_1)...Z(\theta_n) : d\theta_1...d\theta_n \qquad (9)$$

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Whereas in the GLZ case the coefficient functions are expressible in terms of mass-shell projections of retarded functions, the coefficient functions in (9) are connected multiparticle formfactors

$$\langle \Omega \left| A \right| p_n, ..p_1 \rangle^{in} = a_n(\theta_1, ...\theta_n), \quad \theta_n > \theta_{n-1} > ... > \theta_1 \tag{10}$$
$$^{out}\langle p_1, ..p_l \left| A \right| p_n, ..p_{l+1} \rangle^{in}_{conn} = a_n(\theta_1 + i\pi, ...\theta_l + i\pi, \theta_{l+1}, ..\theta_n)$$

which are boundary values of analytic functions in the rapidity variables. If we are interested in operators localized in a double cone $A \in \mathcal{A}(\mathcal{D})$ we should look for the relative commutant $\mathcal{A}(\mathcal{D}) = \mathcal{A}(\mathcal{W}) \cap \mathcal{A}(\mathcal{W}_a)'$ with $\mathcal{D} = \mathcal{W} \cap \mathcal{W}_a'$ and $\mathcal{W}_a$ being the wedge obtained by spatial shifting $\mathcal{W}$ to the right by a. In terms of the above Ansatz this means that the looked for $A's$ should commute with the generators of $\mathcal{A}(\mathcal{W}_a)$ i.e.

$$[A, U(a)\phi(f)U(a)^*] = 0 \tag{11}$$

Since the shifted generators are linear in the Z-F operators and the latter have rather simple bilinear commutation relations, it is possible to solve the recursive relation (the kinematical pole relation) iteratively and characterize the resulting spaces of connected formfactors. Although such recursive formfactor calculations do not reveal existence since bilinear forms (formfactors) need not result from operators, the combination with the previous existence argument would show that the bilinear forms are really particle matrix elements of genuine operators. In the pointlike limit $a \to 0$ the equation characterizes the space of formfactors of pointlike fields. In this case one obtains a basis of this space by invoking the covariance properties of the Lorentz spin. After splitting off a common rather complicated factor shared by all connected formfactors, the remaining freedom is encoded in momentum (rapidity) space polynomial structure which is similar but more complicated than the analogous structure for formfactors of Wick polynomials.

For all QFT which are not factorizing (i.e. in particular for higher dimensional theories) there are no PFGs which generate wedge algebras. In this case the idea would be to try some kind of perturbation theory. A scenario for such a construction may look as follows. Starting in zeroth order with generators which are linear in the incoming creation/annihilation operators, one defines first order generators of the commutant (localized in the opposite wedge) by using the perturbative first order S-matrix in $J = J_0 S^{(1)}_{scat}$. This leads to a first order correction for the coefficient functions of first order generators of the opposite wedge. The hope would be that the imbalance in the commuting property with the original generator would then require a second order correction of S as well as a correction in the coefficient function and that this, similar to the iterative Epstein-Glaser approach for pointlike fields could serve as a perturbative analog of the on-shell bootstrap-formfactor program which bypasses correlation functions of singular fields and leads to a fresh start for a construction program which is also capable to handle the unsolved problem of existence.

In the context of the bootstrap formfactor program for factorizing models one observes an unexpected (and may be even undeserved) simplicity in the

analytic dependence of the formfactors on the coupling strength. There is always a region around zero in which the coupling dependence is analytic. According to general structural arguments there is however no reason that the yet unknown correlation functions will inherit this property. This raises the general question: do on-shell observables have better analytic properties in the coupling than off-shell operators?

It is very interesting to compare the constructive control one has on the basis of the Osterwalder-Schrader setting with that for models constructed in modular bootstrap formfactor program. In the first case the restriction comes from short distance properties; in the almost 40 years history it has not been possible to go beyond superrenormalizable models (mainly $P\phi_2$). On the other hand all the known factorizing models have strictly renormalizable interactions (e.g. the sine-Gordon model interaction is nonpolynomial) and there is no overlap. The weakness of one construction method is the strength of the other. If one could break the limitation set by factorizability as indicated above then the constructive approach would change in favor of the modular wedge generator approach.

Factorizing models are very closely related to chiral conformal theories which "live" (in the sense of modular localization) on a (compactified) lightray. On the one hand there is the general relation of a QFT to its scale invariant short distance limit. Many different massive theories have the same critical limit i.e. belong to one *short distance universality class*. If one only looks at factorizing models than Zamolodchikov has presented conditions under which one can invert this relation in a formal setting of a perturbed conformal theory [43]. On the other hand there is a conceptually quite different relation between d=1+1 massive theories to their chiral *holographic projection [44]*. In that relation the algebraic substrate and the Hilbert space in which it is represented remains unchanged and only the spacetime indexing of the algebras is radically changed in a way that cannot be encoded in a simple geometric relation between the chiral fields on the lightray. But different from the AdS-CFT holography, the *lightfront holography is also a class property* i.e. without enlargement of the Hilbert space there are many ambient theories which are holographic inverses. Only if one had the luck to find generators of the holographic projection which are covariant under the ambient Poincaré group, as it is the case with the Z-F generators in factorizing models, the holographic inverse is uniquely fixed. The rather complicated connection between pointlike generators of the ambient algebra and those of its holographic projection prevent an understanding of this relation by a straightforward inspection.

4 A modular analog of O-S setting and of the Nelson-Symanzik duality

In those cases where the Schwinger functions associated with the O-S Euclideanization admits a stochastic interpretation in the sense of Nelson and

Symanzik, one observes a very strong analogy to the modular localization as will be explained in the following.

The issue of understanding Euclideanization in chiral theories became particularly pressing after it was realized that Verlinde's observation[11] is best understood by making it part of a wider investigation involving angular parametrized thermal n-point correlation functions of observable fields Φ_i in the superselection sector ρ_α

$$\langle\Phi(\varphi_1,..\varphi_n)\rangle_{\rho_\alpha,2\pi\beta_t} := tr_{H_{\rho_\alpha}} e^{-2\pi\beta_t\left(L_0^{\rho_\alpha}-\frac{c}{24}\right)}\pi_{\rho_\alpha}(\Phi(\varphi_1,..\varphi_n)) \qquad (12)$$

$$\Phi(\varphi_1,..\varphi_n) = \prod_{i=1}^{n}\Phi_i(\varphi_i)$$

$$\langle\Phi(\varphi_1,..\varphi_n)\rangle_{\rho_\alpha,2\pi\beta_t} = \langle\Phi(\varphi_n+2\pi i\beta_t,\varphi_1,..\varphi_{n-1})\rangle_{\rho_\alpha,2\pi\beta_t}$$

where the first line defines the angular thermal correlations in terms of a L_0-Gibbs trace at inverse temperature $\beta = 2\pi\beta_t$ on observable fields in the representation π_{ρ_α}. Gibbs states are special (unnormalized) KMS states i.e.states whose correlations fulfill the analytic property in the third line. Their zero point function which is the Gibbs trace of the identity, defines the L_0- partition functions. In contrast to the previously used ground states such thermal correlations are *independent on the particular localization of charges locρ_α*. This is the result of the unitary invariance of the trace and consequently they only depend on the equivalence class i.e. on the sector $[\rho_\alpha] \equiv \alpha$, which makes them valuable objects to study the sector structure (classes of inequivalent representations of the observable algebra). These correlation functions[12] fulfill the following amazing thermal duality relation

$$\langle\Phi(\varphi_1,..\varphi_n)\rangle_{\alpha,2\pi\beta_t} = \left(\frac{i}{\beta_t}\right)^a \sum_\gamma S_{\alpha\gamma}\left\langle\Phi(\frac{i}{\beta_t}\varphi_1,..\frac{i}{\beta_t}\varphi_n)\right\rangle_{\gamma,\frac{2\pi}{\beta_t}} \qquad (13)$$

$$a = \sum_i dim\Phi_i$$

where the right hand side formally is a sum over thermal expectation at the inverse temperature $\frac{2\pi}{\beta_t}$ at the analytically continued pure imaginary angles scaled with the factor $\frac{1}{\beta_t}$. The multiplicative scaling factor in front which depends on the scaling dimensions of the fields Φ_i is just the one which one would write if the transformation $\varphi \to \frac{i}{\beta_t}\varphi$ were an ordinary conformal transformation law. Before presenting a structural derivation of this relation the reader should notice the analogy with the thermal version of the Nelson-Symanzik for massive two-dimensional theories (second section). Since chiral theories are localized on the (compactified) lightray, the analog of the Euclidean spacetime interchange consists of an interchange of the angle with its imaginary version; the stretching

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[11]Verlinde discovered a deep connection between fusion rules and modular transformation properties of characters of rational irreducible representations of chiral observable algebras.

[12]The conformal invariance actually allows a generalization to complex Gibbs parameters τ with $Im\tau = \beta$ which is however not neede in the context of the present discussion.

factor $\frac{1}{\beta_t}$ together with the inverse temperature corresponds to the interchange of the two periodicities in the N-S duality. The appearance of the linear combination of all (finite in rational models) superselection sectors weighted with the Verlinde matrix S has no counterpart in the N-S setting. In simple models as e.g. the multi-component abelian current model [44] the proof of the temperature duality relation can be reduced to properties of the Dedekind eta-function, the Jacobi Θ-functions as well as the Poisson-resummation property. The Kac-Peterson-like character relations

$$\chi_\alpha(\tau) = \sum_\beta S_{\alpha\beta}\chi_\beta(-\frac{1}{\tau}), \ \chi_\alpha(\tau) \equiv tr_{H_{\rho_\alpha}} e^{-2\pi\beta_t\left(L_0^{\rho_\alpha} - \frac{c}{24}\right)} \mathbf{1} \tag{14}$$

is a special case of the (13) when one uses instead of fields the identity. The relevant Verlinde matrix S is the one which diagonalizes the Z_N lattice fusion rules and together with a certain diagonal matrix T generates a unitary representation of the modular group $SL(2,\mathbb{Z})$ whose generators are T: $\tau \to \tau + 1$ and S: $\tau \to -\frac{1}{\tau}$ But a profound understanding of its content can only be achieved by a general structural argument. Under certain technical assumption within the setting of vertex operators[13], Huang recently presented a structural (model-independent) proof [45] of the character relation with Verlinde's definition of S. Huang's proof does not really reveal the deep local quantum physical principles which the analogy to the N-S duality suggests.

The fact that the character relation is a special case of a relation which involves analytic continuation to imaginary rotational lightray coordinates suggests that one should look for a formulation in which the rotational Euclideanization has a well-defined operator-algebraic meaning. On the level of operators a positive imaginary rotation is related to the Moebius transformation $\tilde\Delta^{it}$ with the two fixed points $(-1, 1)$ via the formula

$$e^{-2\pi\tau L_0} = \Delta^{\frac{1}{4}}\tilde\Delta^{i\tau}\Delta^{-\frac{1}{4}} = \tilde\Delta_c^{i\tau} \tag{15}$$

where Δ^{it} and $\tilde\Delta^{it}$ represents the $SL(2,R)$ Moebius subgroups with fixpoints $(0, \infty)$ resp. $(-1, 1)$ and $\tilde\Delta_c^{i\tau}$ the $SU(1,1)$ subgroup with $z = (e^{-i\frac{\pi}{2}}, e^{i\frac{\pi}{2}}) = (-i, i)$ being fixed (the subscript c denotes the compact picture description). Note that $Ad\Delta^{\frac{1}{4}}$ acts the same way on $\tilde\Delta^{i\tau}$ as the Cayley transformation AdT_c, where the T_c is the matrix which represents the fractional acting Cayley transformation

$$T_c = \frac{1}{\sqrt{2}}\begin{pmatrix} i & 1 \\ -i & 1 \end{pmatrix} \tag{16}$$

Ignoring for the moment domain problems for $\Delta^{\frac{1}{4}}$, the relation (15) gives an operator representation for the analytically continued rotation with positive

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[13]The Vertex framework is based on pointlike covariant objects, but unlike Wightman's formulation it is not operator-algebraic i.e. the star operation is not inexorably linked to the topology of the algebra as in C^*algebras of quantum mechanical origin. Although it permits a generalization beyond two dimensions [46], the determination of classifications and representations of higher-dimensional vertex-algebras remains an open problem.

imaginary part $(t > 0)$ in terms of a Moebius transformation with real rapidity parameter. If we were to use this relation in the vacuum representation for products of pointlike covariant fields Φ where the spectrum of L_0 is nonnegative, we would with obtain with $\Phi(t) = e^{2\pi i t L_0}\Phi(0)e^{-2\pi i t L_0}$

$$\langle\Omega\,|\Phi_1(it_1)..\Phi_n(it_n)|\,\Omega\rangle^{ang} = \langle\Omega\,|\Phi_1(t_1)_c..\Phi_n(t_n)_c|\,\Omega\rangle^{rap} \qquad (17)$$
$$= \omega_{2\pi}(\Phi_1(t_1)_c..\Phi_n(t_n)_c)^{rap}$$

The left hand side contains the analytically continued rotational Wightman functions. As a result of positivity of L_0 in the vacuum representation this continuation is possible as long as the imaginary parts remain ordered i.e. $\infty > t_1 > ... > t_n > 0$. On the right hand side the fields are at their physical boundary values parametrized with the rapidities of the compact $\tilde{\Delta}_c^{it}$ Moebius subgroup of $SU(1,1)$. Note that this rapidity interpretation implies a restriction since the rapidities associated with $x = th\frac{t}{2}$ cover only the interval $(-1,1)$. The notation in the second line indicates that this is a KMS state at modular temperature $\beta_{mod} = 1$ ($\beta_{Hawking} = 2\pi\beta_{mod} = 2\pi$) in agreement with the well-known fact that the restriction of the global vacuum state to the interval (-1,1) becomes a state at fixed Hawking-Unruh temperature 2π. Note that only the physical right hand side is a Wightman *distribution in terms of a standard $i\varepsilon$ boundary prescription*, whereas the left hand side is an *analytic function (i.e. without any boundary prescription)*. This significant conceptual (but numerical harmless) difference is responsible for the fact that in the process of angular Euclideanization of chiral models *the KMS condition*[14] *passes to a periodicity property and vice versa.*

The analogy with the generalized Nelson-Symanzik situation suggests to start from a rotational thermal representation in the chiral setting. For simplicity let us first assume that our chiral theory is a model which possess besides the vacuum sector no other positive energy representations. Examples are lattice extension of multicomponent Weyl algebras with selfdual lattices (e.g. the moonshine lattice). In this case $S = 1$ in the above matrix relation (13). Assume for the moment that the Gibbs temperature is the same as the period namely $\beta_{mod} = 1$. According what was said about the interchange of KMS with periodicity in the process of angular Euclideanization we expect the selfdual relation

$$\langle\Omega_1|\Phi(it_1)...\Phi(it_n)|\Omega_1\rangle^{rot} = (i)^{ndim\Phi}\,\langle\Omega_1^E|\,\Phi^E(t_1)...\Phi^E(t_n)\,|\Omega_1^E\rangle^{rot} \qquad (18)$$
$$\langle\Omega_1|\Phi(t_1)...\Phi(t_n)|\Omega_1\rangle^{rot} \equiv tr(\Omega_1,\Phi(t_1)...\Phi(t_n)\Omega_1),\ \Omega_1 \equiv e^{-\pi L_0}$$
$$\Phi^E(t_1)^\dagger \equiv \tilde{J}\Phi^E(t_1)\tilde{J} = \Phi^E(-t_1)^*,\ \left[\tilde{J},L_0\right] = 0$$

where the analyticity according to a general theorem about thermal states [47][48] limits the $t's$ to the unit interval and requires the ordering $1 > t_1 > ... >$

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[14]Contrary to popular believes KMS is not equivalent to periodicity in time but it leads to such a situation if the the involved operators commute inside the correlation function (e.g. spacelike separated observables).

$t_n > 0$. Thermal Gibbs states are conveniently written in the Hilbert space inner product notation with the help of the Hilbert-Schmidt operators $\Omega_1 \equiv e^{-\pi L_0}$, in which case the modular conjugation is the action of the Hermitian adjoint operators from the right on Ω_1 [19]. Since the KMS and the periodicity match crosswise, the only property to be checked is the positivity of the right hand side i.e. that the correlations on the imaginary axis are distributions which fulfill the Wightman positivity. Here the label E on $\Phi(t_1)$ denotes the Euclideanization. For this we need the star conjugation associated with $\tilde{J}$ which interchanges the right with the left halfcircle which because of $L_0 = H + \tilde{J}H\tilde{J}$ commutes with L_0. In that case the modular group of $\Phi^E(t_1) = \Phi(it)$ is $e^{-2\pi t L_0}$ and the modular conjugation is the Ad action of $\tilde{J}$ which changes the sign of t as in the third line (18). Whereas the modular conjugation in the original theory maps a vector $A\Omega_1$ into $\Omega_1 A^*$ with the star being the Hermitean conjugate, the Euclidean modular conjugation is $A^E \Omega_1^E \to \Omega_1^E \tilde{J} A^E \tilde{J} \equiv \Omega_1^E \left(A^E\right)^\dagger$. This property is at the root of the curious selfconjugacy (18). .

There are two changes to be taken into consideration if one passes to a more general situation. The extension to the case where one starts with a β Gibbs state which corresponds in the Hilbert-Schmidt setting to $\Omega_\beta = e^{-\pi\beta L_0}$ needs a simple rescaling $t \to \frac{1}{\beta}t$ on the Euclidean side in order to maintain the crosswise correspondence between KMS and periodicity. Since the Euclidean KMS property has to match the unit periodicity on the left hand side, the Euclidean temperature must also be $\frac{1}{\beta}$ i.e. the more general temperature duality reads

$$\langle \Omega_\beta | \Phi(it_1)...\Phi(it_n) | \Omega_\beta \rangle^{rot} = \left(\frac{i}{\beta}\right)^{ndim\Phi} \left\langle \Omega^E_{\frac{1}{\beta}} \right| \Phi^E(t_1)...\Phi^E(t_n) \left| \Omega^E_{\frac{1}{\beta}} \right\rangle^{rot} \quad (19)$$

The positivity argument through change of the star-operation remains unaffected. This relation between expectation values of pointlike covariant fields should not be interpreted as an identity between operator algebras. As already hinted at the end of section 2 one only can expect a sharing of the analytic core of two different algebras whose different star-operations lead to different closure. In particular the above relation does not represent a symmetry in the usual sense.

The second generalization consists in passing to generic chiral models with more superselection sectors than just the vacuum sector. As usual the systems of interests will be rational i.e. the number of sectors is assumed to be finite. In that case the mere matching between KMS and periodicity does not suffice because all sectors are periodic as well as KMS and one does not know which sectors to match. A closer examination (at the operator level taking the Connes cocycle properties versus charge transportation around the circle into account) reveals that the statistics character matrix S [49] enters as in (13) as a consequence of the well-known connection between the *invariant* content (in agreement with the sector $[\rho]$ dependence of rotational Gibbs states) of the circular charge transport and the statistics character matrix [50][51]. For those known rational models for which Kac-Peterson characters have been computed,

this matrix S turns out to be identical to the Verlinde matrix S which diagonalizes the fusion rules [52] and which together with a diagonal phase matrix T generates a unitary representation of the modular group $SL(2, Z)^{15}$. Confronting the previous zero temperature situation of angular Euclidean situation with the asymptotic limit of the finite temperature identity, one obtains the Kac-Wakimoto relations as an identity between the temperature zero limit and the double limit of infinite temperature (the chaos state) and short distances on the Euclidean side.

This superselection aspect of angular Euclideanization together with the problem in what sense this modular group $SL(2, Z)$ can be called a new symmetry is closely related to a more profound algebraic understanding of the relation between the analytic cores of the two algebras and requires a more thorough treatment which we hope to return to in a separate publication.

Modular operator theory is also expected to play an important role in bridging the still existing gap between the Cardy [54] Euclidean boundary setting and those in the recent real time operator algebra formulation by Longo and Rehren [55]

5 Open problems, concluding remarks

The comparision of the constructive results obtained in the O-S setting and in the old bootstrap-formfactor approach built on the Smirnov construction recipes with the more recent constructions based on modular localization theory gives rise to a wealth of unsolved basic problems of QFT. Here are some of them.

- The O-S formulation and the modular setting are related in a deep and yet mostly unknown way. In order to learn something about this connection one may start with the Wigner representation setting. Recently Guerra has spelled out what the O-S Euclideanization means in the simplest context of the spinless one-particle Wigner space [56]. On the other hand all problems concerning the modular localization setting have been explicitly answered for all positive energy representations [27]. It would be very interesting to translate these results into the O-S setting.

- The old bootstrap dream remained unfulfilled beyond factorizing, and the new modular setting not only explains why a general pure S-matrix approach is not feasible but also indicates that if one views the S-matrix construction as part of a wider framework which aims at generators of wedge localized algebras, this dream still may find its realization in a new construction of QFT which bypasses correlation functions of (necessarily singular) correlation functions of pointlike generators. With new hindsight

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[15]Whereas relativistic causality already leads to an extension of the standard KMS β-strip analyticity domain to a β-tube domain [53], conformal invariance even permits a complex extension of the temperature parameter to τ with $Im\tau > 0$. For this reason the chiral theory in a thermal Gibbs state can be associated with a torus in the sense of a Riemann surface, but note that in *no physical sense* of localization this theory lives *on* a torus.

and a new conceptual setting one should revisit the properly re-formulated old problems.

- The d=1+2 massive Wigner representation can have anomalous (not semi-integer) spin which leads to plektonic (braid group) statistics. The simplest abelian family of representations is that of Z_N-anyons. Such representations activate representations of the Poincaré group in which the Lorentz part is represented through the Bargmann covering $\widetilde{SO(2,1)}$. The modular theory of these string-like representations has been worked out in [57] and it is known that d=1+2 anomalous spin representations are the only Wigner representations whose associated QFT has vacuum-polarization which prevent the standard on-shell free field realization [36]. It would be very interesting to understand how an O-S like Euclidean formulation would look like.

- Up to now models of QFT have been "baptized" and studied in the setting of Lagrangian quantization (either canonical of functional integral). More recently the bootstrap-formfactor setting led to models which do not possess a Lagrangian description (e.g. the Z_N model in [60] whose natural description is in terms of Z_N braid group statistics). There are indications that interactions in terms of string-localized fields (which apparently do not permit a Euler-Lagrange characterization) extends the possibilities for formulating interactions. The ghostfree potential for the physical fields of zero mass finite helicity representations (e.g. the vectorpotential associated with the electromagnetic field) are necessarily string-localized [27]. Also in this case it should be possible to use these objects outside the Lagrangian framework directly in the implementation of interactions. Such a description would be particularily interesting for higher helicities as in the case of the graviton. An intrinsic description of QFT "without the classical crutches" of Lagrangian quantization is an old dream of Pascual Jordan, the protagonist of "Quantelung der Wellenfelder". The continuation of the ongoing attempts may still lead to a fulfillment of this dream.

- The relation between heat bath thermal behavior and the purely quantum thermal manifestations of vacuum polarization (Hawking, Unruh, Bekenstein) have received a lot of recent attention, but they still have not been adequately understood. Modular properties (especially the split property), analytic continuation and Euclideanization are expected to play an important role.

Quantum field theory, which in the frame of mind of some string theorist has become a historical footnote of their theory, had already been declared dead on two previous occasions; first in the pre-renormalization ultraviolet crisis of the 30s and then again by the protagonists of the S-matrix bootstrap in the 60s. But each time a strengthened rejuvenated QFT re-appeared. It is clear that an area which still produces many fundamental new questions is very far from its closure.

There is however a new sociological problem which poses a serious impediment to the kind of physics as it developed over several centuries through rational discourse and which does not necessarily aim at a "theory of everything". Since the times of Galilei and Newton its aim was the de-mystification of nature and in this role it had an enormous impact on the European enlightenment and more general on western civilization. This is presently threatened by a new trend of re-mystification. Whereas this is most visible in the Kulturkampf which the movement of Intelligent Design in the US unleashed against Darwin's formation of species, the trend of re-mystification has already entered particle physics. When one of the most influential particle physicists invites (without any irony) the physics community to interpret the meaning of the big letter "M" in M-theory as "Mystery" the aims have already been redefined and the new direction certainly is not of a critical discourse. It is also plainly visible in recent outings by well-known string theorists [58] and the question of whether anthropic arguments are camouflaged Intelligent Design arguments is somewhat academic as far as the future of particle physics is concerned since hegemonic aspects of string theory ("There is no other Game in Town" [59]) are even more detrimental to the fundamental research in particle physics than the imposition of religious beliefs which does not have a direct impact on the substance of research.

It is naive to believe that in times of globalization there can be any area of human activities which can be kept protected from the intellectual and material arrogance of the new Zeitgeist of the post cold war era. A *Hegemon* will not change the established terminology but he can and does re-define its meaning. The concepts of human rights are not abandoned, they are just redefined to suit the Hegemon.

The present crisis in particle physics is not an isolated passing event, it has a solid material basis. The hegemonial tendency in physics does not favor conceptual progress through the dialectic sharpening of contradictions and antinomies with the existing principles with the aim to reach a breaking point from where a new principle could take over. The market forces rather favor the much faster path to personal fame by contributions which basically consists in the acclamation of prevailing fashions. Through the hegemonial rein of the market the possibility that somebody in an old-fashioned patent office will have the productive leisure to follow his own innovative ideas is practically ruled out; this only remains as a nostalgic picture. The modern role model of a particle physics theoretician is rather that of a citation-supported young star who tries to stay on top of fashions by following all updates of big Latin Letters. He is always prepared to support any change by cranking out new computations and leaving aside any conceptual confrontation with traditional principles for which his training in any case would be insufficient. A novice in QFT would endanger his career[16] if instead of the fast calculational entrance into one of the

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[16]A warning example for young physicists that straying away too much from dominating fashions may wreck an academic career is the fate of H.-W. Wiesbrock, who a short time after his innovative work on the application of modular operator theory in particle physics at the FU did not find any place to continue his career.

ongoing globalized fashions he would choose the more arduous path of getting to the conceptual and mathematical roots of a problem, including some at least rudimentary knowledge about its history. This traditional path was still possible for the generation which includes Karowski and Schrader (and myself); in those days there were fashions, but they they did not yet grow into hegemonially managed monocultures. Nowadays it is very easy to compile a list of "who is who" in the administration of the particle physics crisis. One only has to look at the list of the editorial board of recently founded journals, a particular valid illustration is JHEP. With such powerful control over the globalized impact of papers and the academic market it is clear that this is not going to be a transitory phenomenon of short duration.

Sociologists and historians of physics have tried to analyze the amazing progress which took place in war-torn Europe (in particular Germany) in the aftermath of world war 1. Some have attributed the loss of certainty in favor of probabilistic concepts in the emerging quantum theory to the gloom and doom Zeitgeist [61] (which found its expression in the widely red historical treatise "The Decline of the West" by Oswald Spengler) from where, so they argue, one could become part of the avant-garde only by a very revolutionary tabu-breaking conceptual step. Such explanations do not appear very plausible. It should be considerably more natural to explain the present crisis in terms of all-pervading rule of the globalized market in which impact parameter and personal fame (and not the gain of genuine knowledge) are the propelling forces.

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